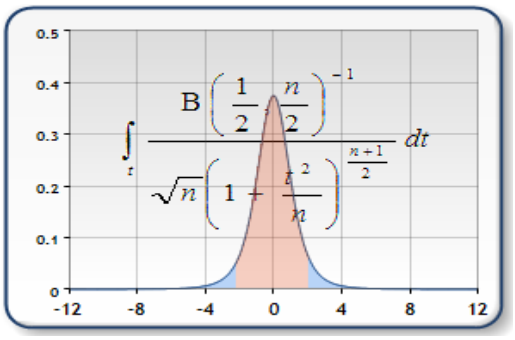
Extended Statistics HP-41 Module



Overview

Most of the functions in this ROM were included in the former version of the "Curve Fitting & Statistics" Module. The changes made to the Curve Fitting functionality in that module were also the perfect excuse to separate the statistics contents into its own ROM, dedicated to statistics and probability.

Some functions are also in the SandMath, a real Math powerhouse – but a few ones have been written anew; and yet more new functions have been added as result of the collaboration with Greg McClure on his GJM Module.

This module has three distinct sections. The first one includes the more fundamental functions covering sums, means, and other basic numeric calculations - such as linear regression, combinations/permutations, etc. Then it moves into the distributions section where you can find functions to calculate the density and probability functions for the most important distributions. Finally a third section includes the four Primality functions as well as other auxiliary functions and utilities related with the topic at hand.

Without further ado, let's see the functions included in the module. Refer to the individual function descriptions later on for details on the syntax and use instructions – always a tricky think when it comes to the statistics topics.

XROM	Function	Description	Input	Author
06,00	**-X STAT 1E**	***Section Header***	***n/a***	--
06,01	**%T**	Total percentage	y,x in Y,X	*Poul Kaarup*
06,02	**Σ0**	Sum of mantissa digits	value in X	*Ángel Martin*
06,03	**Σ1**	Sum of N integers	N in X	*Poul Kaarup*
06,04	**Σ1/N**	Harmonic Number	n in X	*Ángel Martin*
06,05	**Σ2**	Sum of squares of N integers	N in X	*Poul Kaarup*
06,06	**Σ3**	Sum of cubes of N Integers	N in X	*Poul Kaarup*
06,07	**ΣX^N**	Geometric Sums	N in Y, X in X	*Ángel Martin*
06,08	**ΣRG?**	Stat Reg Location	none	*Ken Emery*
06,09	**AGM**	Arithmetic-GeometricMean	x,y in Stack	*Ángel Martin*
06,10	**AMEAN**	Arithmetic Mean	bbb.eee in X	*Ángel Martin*
06,11	**CNK**	Combinations	arguments in X,Y	*Ángel Martin*
06,12	**CORR**	LR Correlation	Σ+ Data Set	*JM Baillard*
06,13	**COV**	LR Covariance	Σ+ Data Set	*JM Baillard*
06,14	**EVEN?**	Tests for even value	value in X	*Ángel Martin*
06,15	**GCD**	Greater Common Divisor	arguments in X,Y	*Ángel Martin*
06,16	**GHM**	Geometric-Harmonic Mean	x,y in Stack	*Greg McClure*
06,17	**GMEAN**	Geometric Mean	bbb.eee in X	*Ángel Martin*
06,18	**HMEAN**	Harmonic Mean	bbb.eee in X	*Ángel Martin*
06,19	**LCM**	Least Common Multiple	arguments in X,Y	*Ángel Martin*
06,20	**L1**	Shows line equation in ALPHA	a,b in Stack	*Ángel Martin*
06,21	**LR**	Linear Regression	Σ+ Data Set	*JM Baillard*

06,22	**LRX**	LR X-Value	Σ+ Data Set, intercept	*Ángel Martin*
06,23	**LRY**	LR Y-Value	Σ+ Data Set, abscissa	*JM Baillard*
06,24	**ODD?**	Tests for odd value	value in X	*Ángel Martin*
06,25	**PMEAN**	Generalized Power Mean	p in Y; bbb.eee in X	*Ángel Martin*
06,26	**PNK**	Permutations	arguments in X,Y	*Ángel Martin*
06,27	**RCLΣ**	Stat Reg to Stack	none	*JM Baillard*
06,28	**STLINE**	Straight Line equation	(x1,y1) & (x2,y2) in stack	*Ángel Martin*
06,29	**ST<>Σ**	Swap Stack & Stat Registers	none	*Nelson F. Crowle*
06,30	**-DISTRIBTN**	***Section Header***	***n/a***	**--**
06,31	**BIN**	Binomial Distribution	Prompted by program	*Ángel Martin*
06,32	***"BNP"***	Binomial P(n,p,x)=k	n,x in Y, X	*JM Baillard*
06,33	***"BNP+"***	Binomial P(n,p,x)<=k	n,k,x in stack	*JM Baillard*
06,34	**C2CP**	Chi-Square Probability Fnc.	n,x in Y, X	*JM Baillard*
06,35	**C2DF**	Chi-square Density Function	n,x in Y, X	*Ángel Martin*
06,36	**CHI2**	Chi^2 Distribution Driver	prompts for u value	*HP Co. (Stat Pac)*
06,37	***"FDST"***	Snedecor's F-Distribution Driver	menu: : "N1 N2 Q"	*HP Co. (Stat Pac)*
06,38	**FSDF**	Snedecor's F Density Function	Z: N1. Y: N2, X: Point	*Eugenio Úbeda*
06,39	***"FSCP"***	Snedecor's F Probability Fnc.	n,x in Y, X	*Eugenio Úbeda*
06,40	**NRDF**	Normal Density (m,s)	Z: Mean, Y: Sdev, X: Point	*Ángel Martin*
06,41	**NRCP**	Normal Probability (m,s)	Z: Mean, Y: Sdev, X: Point	*Ángel Martin*
06,42	**POIS**	Poisson Distribution	prompts for values	*Ángel Martin*
06,43	**PSDF**	Poisson Density Function	Y: Mean, X: #Events	*Ángel Martin*
06,44	***"PX"***	Standard Normal Probability	Argument in X	*Mike (Stgt)*
06,45	***"PXX"***	Probability between -x and x	Argument in X	*Mike (Stgt)*
06,46	**QNTL**	Quantile - inverse of NRCP	Argument in X	*Ángel Martin*
06,47	***"TDIST"***	T-Distribution Driver program	Menu driven	*HP Co. ??*
06,48	**TSDF**	Student's T Density Function	N in Y, Point in X	*Eugenio Úbeda*
06,49	**TSCP**	Student's T Probability Func.	n,x in Y, X	*Eugenio Úbeda*
06,50	***"XP"***	Inverse of PX	Argument in X	*Mike (Stgt)*
06,51	***"XXP"***	Inverse of PXX	Argument in X	*Mike (Stgt)*
06,52	**-PRIMALITY**	***Section Header***	***n/a***	**--**
06,53	**1/GM**	Inverse Gamma Function	Argument in X	*JM Baillard*
06,54	**BETA**	Beta Function	Arguments in X,Y	*JM Baillard*
06,55	**D%**	Difference Percent	x,y in Stack	*Ángel Martin*
06,56	**ERF**	Error Function	x in X	*Martin - Baillard*
06,57	**GMF**	Gamma Function	Argument in X	*JM Baillard*
06,58	**NXTPRM**	Next Prime	value in X	*PoulKaarup*
06,59	**PFACT**	Prime Divisor (factor)	Argument in X	*Peter Platzer*
06,60	**PTWIN**	Next Twin Primes	Argument in X	*Peter Platzer*
06,61	**PRIME?**	Prime Test	Argument in X	*Poul Kaarup*
06,62	**RAND**	Random Number (w/ Timer)	none	*JM Baillard*
06,63	***"SLV"***	Solve Subroutine	x1, x2 in X,Y, fname in Alpha	*PPC Members*

Note that this module was designed to be self-contained, i.e. there are no dependencies on any other one, not even the Library#4 (therefore never mind if you never heard of that one before). All required auxiliary functions for the FOCAL programs are included in the third section.

1 – Basic Statistical Functions.

Summation Functions.{ Σ0, Σ1, Σ2, Σ3, Σ1/N, ΣX^N }

Use them to calculate the resulting sum of a finite series of integer numbers (Triangular Numbers), their squares (Squared Pyramidal Numbers), cubes (Squared Triangular Numbers), or generalized (Faulhaber's) powers. Also included is the harmonic number H(n), which sums the reciprocal of the numbers – i.e. a particular case of the generalized case with exponent -1.

The formulas for the first three cases are as follows:

$$T_n = \sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$P_n = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$$

- **Σ0** sums the mantissa digits of the number in X. For example: PI, **Σ0** => 40

- **Σ1, Σ2, Σ3** calculate the first three cases using the explicit formulas – much faster than performing the actual summation even for short series.

- **Σ1/N** calculates the Harmonic number of the argument in X; that is the sum of the reciprocals of the natural numbers (which excludes zero) lower and equal to n. It is used in the calculation of numerous special functions, like the Kelvin and the Bessel functions of the second kind, K(n,x) and Y(n,x).

  $$H_n = \sum_{k=1}^{n} \frac{1}{k}.$$

  Example: calculate H(5) and H(25).

  5, [**Σ1/N**] => 2.283333333
  25, [**Σ1/N**] => 3.815958178

- **ΣN^X** Calculates a generalized value of the Faulhaber's formula for integer values of x. – The few first integer values of x have explicit formulas for the result, but that's not the case for a general value, which can also be non-integer. Obviously for x=-1 this function returns identical results than the previous one, albeit slower due to the additional complexity of the term.

  Example: Check the triangular (x=1) and pyramidal (x=2) formulas for n=10 – which are particular cases of the Faulhaber's Formula, involving Binomial coefficients and Bernoulli's numbers. See the link below for details: http://en.wikipedia.org/wiki/Faulhaber%27s_formula

  10, ENTER^, 1, [**ΣN^X**] => 55.00000000
  10, ENTER^, 2, [**ΣN^X**] => 385.0000000

Single and Duplex Means (to an end).

In the means department there is a very complete selection of choices: arithmetic, geometric and harmonic means are calculated on a set if data registers controlled by the control word "bbb.eee" in X– i.e. beginning and end registers*, and *not* the statistical registers as defined by* **ΣREG** ! . Also a generalized exponential mean is available using the same syntax.

The **AMEAN**, **GMEAN**, and **HMEAN** functions calculate the means of multiple values stored in data registers. Entering the control word describing the register set in X and executing AMEAN, GMEAN, or HMEAN will result in that mean being put into X (and the control word saved in LastX). So, for example, to get one of these means for values in registers 10 thru 15, put 10.015 in X and execute the appropriate mean function.

But there is more: The **PMEAN** function is also available for a generalized mean function. The power "p" is put into Y and the control word in X, and the Generalized Power Mean is calculated for the values pointed to by the control word. The PMEAN formula is:

$$M_p(x_1, \ldots, x_n) = \left(\frac{1}{n} \sum_{i=1}^{n} x_i^p \right)^{\frac{1}{p}}$$

For p=0 this would normally lead to a problem. However the limit for this expression as p -> 0 yields the Geometric Mean, so when p=0, the GMEAN function code is used.

From the above formula you can see that p=1 yields the Arithmetic mean, and p=-1 yields the Harmonic mean. However fractional and other negative values can be used, and you will notice that as p becomes infinite (positive), the mean tends to be the MAX value of the numbers. As p becomes infinite (negative), the mean tends to be the MIN value of the numbers.

With the exception of the AMEAN program, all values used in the registers must be non-zero positive values. Otherwise a DATA ERROR will occur.

Let's move now to the duplex means on a pair of numbers placed in X and Y registers: the Arithmetic-Geometric mean **AGM** and the Harmonic-Geometric mean **HGM**. An interesting definition of the mean of two values occurs when combining Arithmetic, Geometric, and Harmonic means.

- The Arithmetic-Geometric mean is a special value, defined as the common limit of A=AMean(A,B) and B=GMean(A,B) repeated until A-B = 0.
- The Geometric-Harmonic mean is defined as the limit of A=GMean(A,B) and B=HMean(A,B) repeated until A-B = 0.

AGM calculates the Arithmetic-Geometric Mean, whilst **GHM** calculates the Geometric-Harmonic mean.

As an interesting note, AM(A,B) >= AGM(A,B) >= GM(A,B) >= GHM(A,B) >= HM(A,B).

What happened to the Arithmetic-Harmonic mean? That is simply the Geometric mean in disguise, thus no need for such function. Finally, note that taking p=0.5 in the PMEAN function (on two registers) will NOT yield the AGM (and -0.5 will NOT yield the GHM) unless, of course, the register values are identical! It is not that simple to get those values, and the power value required changes depending on the two values chosen for AGM or GHM.

Combinations and Permutations – two must-have classics.

Nowadays it would be unconceivable to release a calculator without this pair in the function set – but back in 1979 when the 41 was designed things were a little different. So here there are, finally and for the record.

- **PNK** calculates Permutations, defined as the number of possible different arrangements of N different items taken in quantities of K items at a time. No item occurs more than once in an arrangement, and different orders of the same R items in an arrangement are counted separately.The formula is:

$$\frac{n!}{(n-k)!}.$$

- **CNK** calculates Combinations, defined as the number of possible sets or N different items taken in quantities or K items at a time. No item occurs more than once in a set, and different orders of the same R items is a set are not counted separately. The formula is:

$$\frac{n!}{k!(n-k)!}$$

The general operation include the following enhanced features:

- Gets the integer part of the input values, forcing them to be positive.
- Checks that neither one is Zero, and that n>k
- Uses the minimum of {k, (n-k)} to expedite the calculation time
- Checks the Out of Range condition at every multiplication, so if it occurs its determined as soon as possible
- The chain of multiplication proceeds right-to-left, with the largest quotients first.
- The algorithm works within the numeric range of the 41. Example: CNK(335,167) is calculated without problems.
- It doesn't perform any rounding on the results. Partial divisions are done to calculate **CNK**, as opposed to calculating first **PNK** and dividing it by r!

Provision is made for those cases where n=0 and k=0, returning zero and one as respective results. This avoids DATA ERROR situations in running programs, and is consistent with the functions definitions for those singularities.

Note as well that there is no final rounding made to the result. This was the subject of heated debates in the HP Museum forum, with some good arguments for a final rounding to ensure that the result is an integer. The SandMath implementation however does not perform such final "conditioning", as the algorithm used seems to always return an integer already. Pls. Report examples of non-conformance if you run into them.

Example: Calculate the number of sets from a sample of 335 objects taken in quantities of 167:

Type: 335, ENTER^, 167, XEQ "**CNK**" -> 3,0443587 99

Example: How many different arrangements are possible of five pictures, which can be hung on the wall three at a time:

Type: 5, ENTER^, 3, XEQ "**PNK**" -> 60,00000000

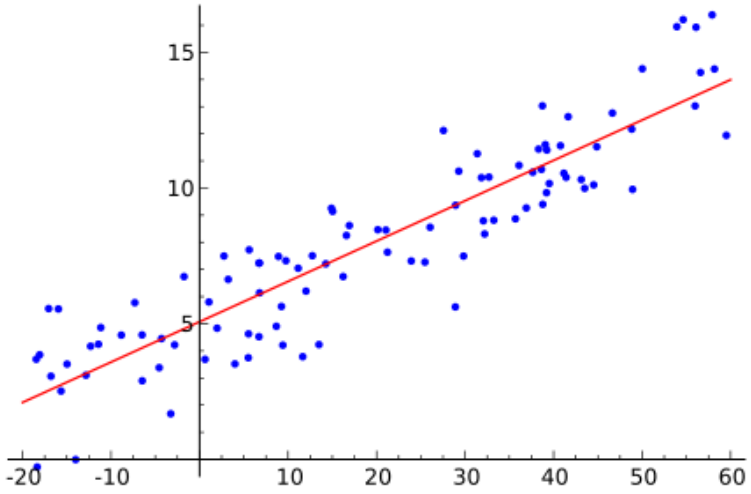
The execution time for these functions may last several seconds, depending on the magnitude of the inputs. The display will show "RUNNING..." during this time.

Linear Regression – Let's not digress.

The following four functions deal with the Linear Regression, the simplest type of the curve fitting approximations for a set of data points. They complement the native set, which basically consists of just **MEAN** and **SDEV**.

	Function	Description	Author
[ΣΣ]	**CORR**	Correlation Coefficient of an X,Y sample	JM Baillard
[ΣΣ]	**COV**	Covariance of an X,Y sample	JM Baillard
[ΣΣ]	**LR**	Linear Regression of an X,Y sample	JM Baillard
	LRX	X-Value for a Y point	Ángel Martin
[ΣΣ]	**LRY**	Y- value for an X point	JM Baillard

Linear regression is a statistical method for finding a straight line that best fits a set of two or more data pairs, thus providing a relationship between two variables. Using the well-known method of least squares, **LR** will calculate the slope A and Y-intercept B of the linear equation: Y = Ax + B.



Results are placed in Y and X registers respectively. When executed in RUN mode the display will show the straight-line equation, similar to the **STLINE** function described before.

Example: find the y-intercept and slope of the linear approximation for the data set given below:

X	0	20	40	60	80
Y	4.63	5.78	6.61	7.21	7.78

Assuming all data pairs values have been entered using Y-value, ENTER^, X-value, Σ+; we type:
XEQ "**LR**" -> 0,038650000 and X<>Y -> 4,856000000 producing the following output in FIX 2:

As to the remaining functions, **COV** calculates the sample covariance. **CORR** returns the correlation coefficient, and **LRY** the linear estimate for the function at the given point.

For the same sample still in the calculator's memory, we obtain the values:

Covariance = 38.65; CORR=0.987954828; LRY=4.894184454 (using Corr value as X)

Ratios and other Tools.

- **%T** and **D%** are miniature functions to calculate the percent of a number relative to another one (its reference), and the delta percent between the numbers in Y(reference) and X(new value). The formulas are:

  %T(y,x) = 100 x / y ; D% = 100 (x-y) /x

  Example: the relative percent of 4 over 25 is 16%.- You type: 25, ENTER^, 4, XEQ "**%T**"
  Example: the delta percent of a change from 85 to 75 is –11,765%

- **GCD** and **LCM** are fundamental functions also inexplicably absent in the original function set. They are short and sweet, and certainly not complex to calculate. The algorithms for these functions are based on the PPC routines **GC** and **LM** – conveniently modified to get the most out of MCODE environment.

  If a and b arenot both zero, the greatest common divisor of a and b can be computed by using least common multiple (lcm) of a and b:

  $$\gcd(a,b) = \frac{a \cdot b}{\operatorname{lcm}(a,b)}.$$

  Examples: GCD(13,17) = 1 (primes), GCD(12,18) = 6; GCD(15,33) = 3
  Examples: LCM (13,17) = 221; LCM(12,18) = 36; LCM(15,33) = 165

- **ST<>Σ** exchanges the contents of five statistical registers and the stack (including L). Use it as a convenient method to review their values when knowing their actual location is not required.

- **RCLΣ** recalls the contents of five statistical registers to the stack (including L). It is therefore just one half of the previous function.

- **ΣRG?** returns to the X register the current pointer to the statistics registers block. It is identical to the function ΣREG? in the 41CX.

- **ODD?** and **EVEN?** are simple tests to check whether the number in X is odd or even. The answer is YES / NO, and in program mode the following line is skipped it the test is false. The implementation is based on the MOD function, using MOD(x,2) = 0 as criteria for evenness.

- **RAND** is a random number generator that uses the current time as seed. If the Time Module is not present the function uses the value in X as seed. The result value is within 0 and 1.

- **STLINE** calculates the slope "m" and 0-intercept "p" of a straight line that passes through the points (x1, y1) and (x2, y2) stored in the stack registers. The equation is: Y = m X + p

- **L1** puts the straight line equation in the Alpha register, using the values in X- and Y- as slope and 0-intercept for the line. This function is used internally in the Linear Regression and **STLINE** functions.

Primes and other Relatives.

- **PRIME?** Is a primality test function, which will return YES/NO in run mode and skip a line if false in a running program.

- **NXTPRM** Is a fast and accurate method to obtain the next prime following the value in X (thus not itself in case it already is a prime number).

- **PTWIN** is similar to NXTPRM, only that it'll find the next Twin primes following the value in X. Those are two consecutive prime numbers p1 and p2, such that p2 = (p1 + 2).

- **PFACT** Is a fast and accurate method to obtain prime divisors of the value in X. This function can be used repeatedly to obtain the complete prime factor decomposition, see the short FOCAL program example provided below.

Line	Instruction	Line	Instruction
01	**LBL "PFCTR"**	13	X=Y?
02	CLA	14	GTO 05
03	**AINT**	15	"|-*"
04	"|- ="	16	/
05	LBL 00	17	GTO 00
06	ENTER^	18	LBL 05
07	**PFACT**	19	AVIEW
08	**AINT**	20	END

As an exercise, improve this program so that the repeated factors are grouped in an exponent, as opposed to in the single fashion listing in ALPHA. You can refer to the SandMath and SandMatrix modules for more sophisticated Prime Factorization programs.

2 – Statistical Distributions.

Moving on to the next section, the following pages describe the different statistical distribution included in the module. Some will have individual functions to perform the calculations independently, whilst other programs are of the "driver" type – providing a micro environment to do calculations related to a given distribution.

It is important to note the naming conventions for these functions. In general there are three types:

- Distribution density function – use "**DF**" as last two letters of function name
- Distribution Cumulative Probability – use "**CP**" as last two letters of function name
- Distribution "driver" program - some include the word "DIST" or "DST" in their names

The following table shows a summary of the available functions by type:

Distribution	Density Function	Cumulative Probability	Driver
Normal	**NRDF**	**NRCP**	**--**
T-Student	**TSDF**	**TSCP**	**TDIST**
F-Snedecor	**FSDF**	**FSCP**	**FDIST**
Chi-Squared	**C2DF**	**C2CP**	**CHI2**
Poisson	**PSDF**	**--**	**POIS**
Binomial	**BNP**	**BNP+**	**BIN**

While some of these functions are implemented as MCODE routines, others are a combination of some module functions in short FOCAL "wrappers" – which appear as MCODE functions in the module's FAT and in your own FOCAL code as program steps. Note however that they likely utilize data registers and don't always follow the "argument to LastX" rule due to their hybrid nature.

A number of auxiliary functions are provided to facilitate the calculation of the distributions listed above. The more important ones are the error function erf(x) – implemented in MCODE as a particular case of the hypergeometric function -, as well as Euler's Gamma and Beta functions (and inverse Gamma as well for convenience). These last two use a continuous fractions method for the calculation – slightly faster than the Lanczos formula and for the most part with comparable accuracy.

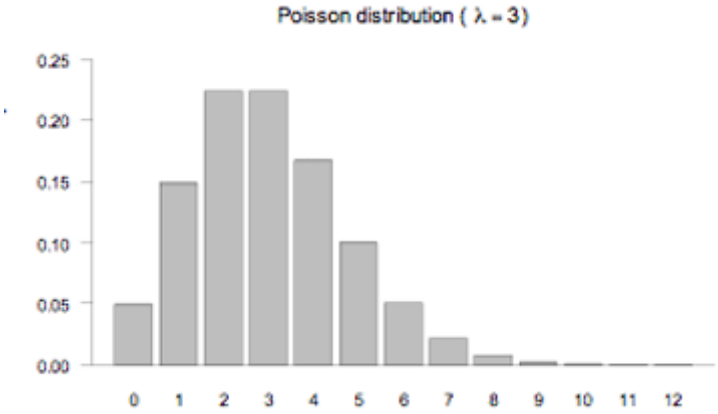
Unfortunately there wasn't enough room left in the module to also include the Incomplete Gamma and Incomplete Beta functions, therefore some approximation formulas are used for the cumulative probability of the T- and F- distributions. Note however that these two functions are included in the SandMath Module in case you need them.

Poisson Standard Distribution.{ PSD , POIS }

PSD is the Statistical function that calculates the Poisson Standard Distribution. In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time and/or space if these events occur with a known average rate and independently of the time since the last event

A discrete stochastic variable X is said to have a Poisson distribution with parameter λ>0, if for k = 0, 1, 2, ... the probability mass function of X is given by:

$$f(k;\lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!},$$



Its inputs are k and λ in stack registers X and Y. **PSDF**'s result is the probability corresponding to the inputs.

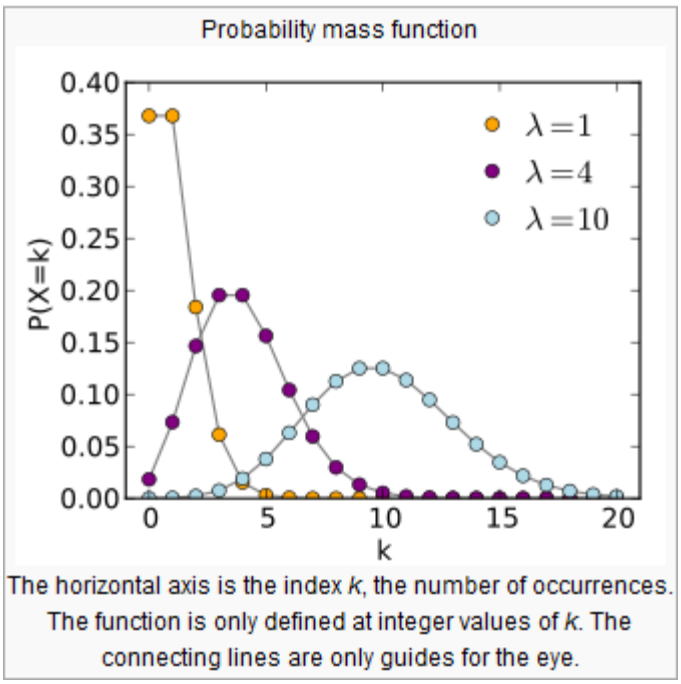
Example 1.-

Calculate the probability mass function for a Poisson distribution with parameters: l=4, k=5

4, ENTER^, 5, **XEQ** "PSD"
Returns: 0.156293452

Example 2: do the same for l=10 and k=10

10, ENTER^, **XEQ** "PSD"
Returns: 0.125110036



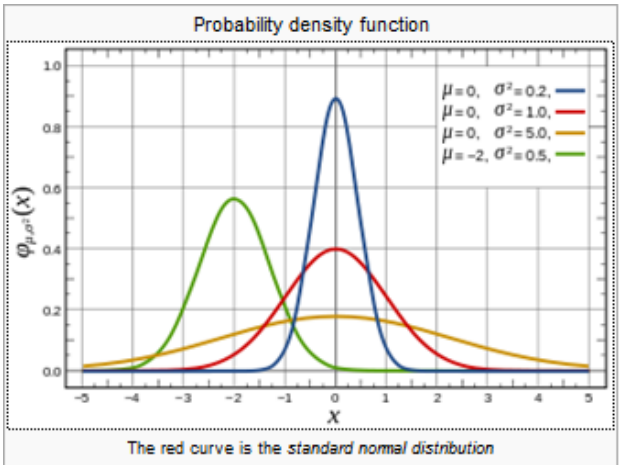
Normal Distribution Functions.{ NRDF , NRCP }

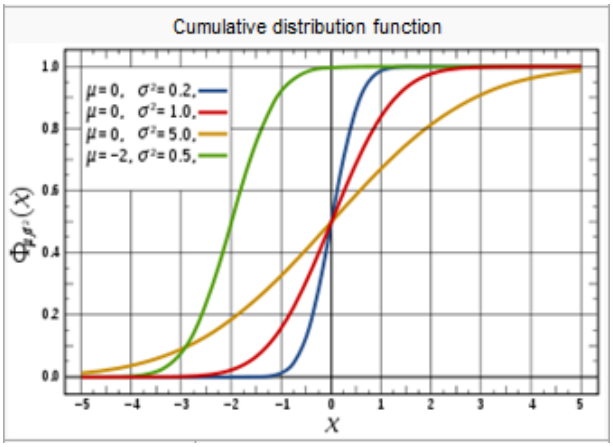
In probability theory, the normal (or Gaussian) distribution is a continuous probability distribution that has a bell-shaped probability density function, known as the Gaussian function or informally as the bell curve:

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

The parameter μ is the mean or expectation (location of the peak) and σ^2 is the variance. σ is known as the standard deviation. The distribution with μ = 0 and σ^2 = 1 is called the standard normal distribution or the unit normal distribution

NRDF expects the mean and standard deviation in the Z and Y stack registers, as well as the argument x in the X register. Upon completion x will be saved in LASTx, and f(μ,σ,x) will be placed in X. It has an all-MCODE implementation, using 13-digit routines for increased accuracy.





The figures above show both the density functions as well as the cumulative probability function for several cases. The Error function **ERF** can be used to calculate the **NRCP** – no need to apply brute force and use **NRDF** in an **INTEG**-like scenario, much longer to obtain or course. The relation to use is:

$$F(x;\,\mu,\sigma^2) = \Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right], \quad x \in \mathbb{R}.$$

Example program: The routine below calculates CPF. Enter μ, σ, and x values in the stack.

```
01  LBL "NRCP"      08  /
02  RCL Z           09  ERF
03  -               11  INCX
04  X<>Y            12  2
05  /               13  /
06  2               14  END
07  SQRT
```

Cumulative Probability Function and its Inverse.{ PX , PXX , XP , XXP , QNTL }

The following routines perform the calculation of the cumulative probability and its inverse, i.e. to obtain the value x that yields a given value for the probability. The inverse functions are based on the Secant or Halley's methods to iterate for the solution.

As a convenience shortcut, functions **PXX** and **XXP** are provided to obtain the CENTRAL probability within a certain zone of the arguments [-x, x] – and the inverse value of X to yield said probability. The expression to obtain this is given by the formula: PXX = 2 P(x) - 1

Note that all these functions *assume a Standard Normal distribution*, with zero mean and standard deviation equal to 1, that is: σ=1 and μ=0

The Secant method is implemented as per the PPC ROM routine **SLV** – also contained in this module. Refer to the PPC ROM manual for details on its operation if used independently from these functions.

Halley's method uses the following expression to calculate the successive approximations to the root:

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2[f'(x_n)]^2 - f(x_n)f''(x_n)}$$

where our function in this case is f(x) = [NRCP(x) – Value], thus we take advantage of the fact:

f '(x) = NRDF and
f "(x) = -k f '(x);

and therefore the general expression above gets simplified considerably since f '(x) is in both numerator and denominator and thus cancelled out.

Examples. Which argument yields a probability of 75% for a Standard Normal distribution?

a) Using **XP**: 0.75, XEQ "XP" -> 0,674489750
b) Using **QNTL**: 0.75, XEQ "QNTL" -> 0,674489750

What is the cumulative probability for the argument obtained in the previous example?

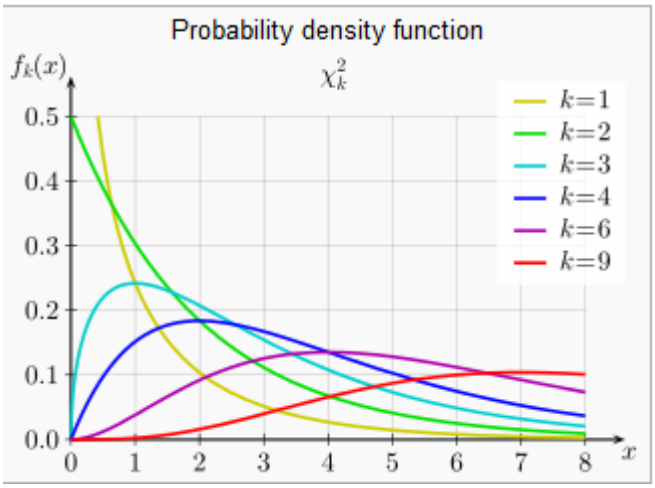
Type: 0, ENTER^, 1, RCL Z, XEQ "NRCP", -> 0,750000000

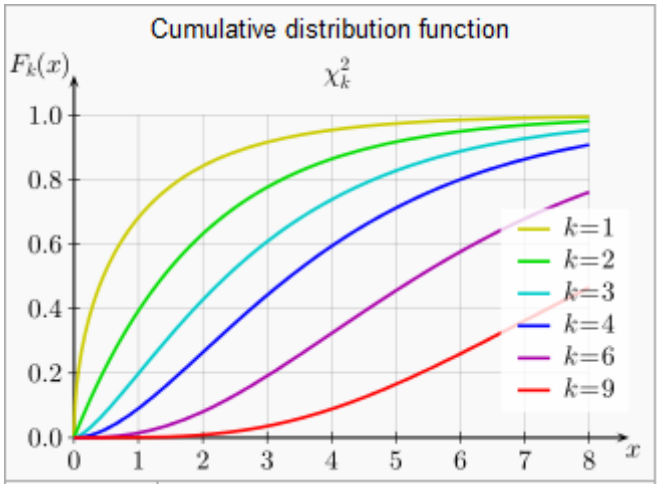
The accuracy is quite good, also holding up well across the entire range of values for both functions – thanks to the iterative methods employed. Execution speed is slightly faster for **XP** than for **QNTL**, but this one is more accurate for arguments in the vicinity of 1.

Note: By comparison with these functions, the SandMath module uses the inverse Error Function **IERF** to calculate the inverse probabilities. That method is much faster and slightly more accurate – but there wasn't enough room left in this module to implement it.

Chi-Squared Distribution { C2DF , C2CP , CHI2 }

In probability theory and statistics, the chi-squared distribution (also chi-square or χ^2-distribution) with k degrees of freedom is the distribution of a sum of the squares of k independent standard normal random variables. It is a special case of the gamma distribution and is one of the most widely used probability distributions in inferential statistics, e.g., in hypothesis testing or in construction of confidence intervals. When it is being distinguished from the more general non-central chi-squared distribution, this distribution is sometimes called the central chi-squared distribution.





There are three functions available in the XMSTAT module dealing with this distribution.

- This first two **C2DF** and **C2CP** are independent implementations for the density and probability functions respectively, using the values in Y (degrees of freedom) and X (argument). The result is placed in X but no LastX is done.

- The third and last **CHI2** is a driver routine to calculate both the density function and the cumulative probability. After prompting for the parameter μ it presents the simple menu of choices shown below:

Simply enter the argument and press [**A**] for the Probability distribution function (same as **CHDF**), or [**B**] for the cumulative distribution function P(x). See the formulas below:

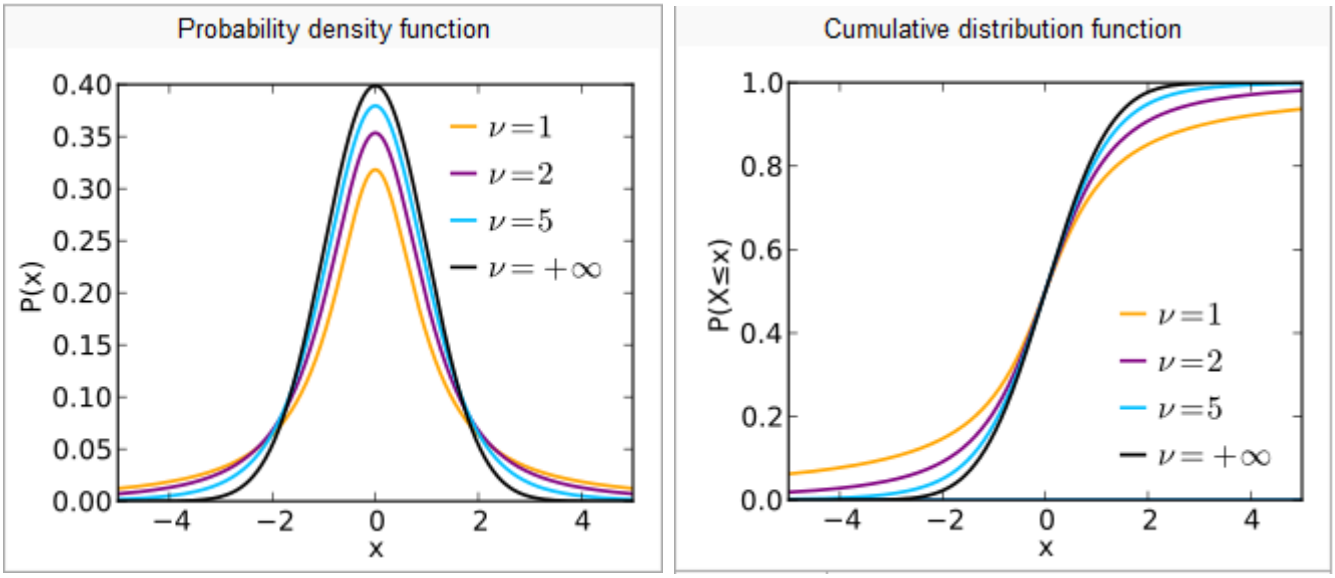
PDF	$\frac{1}{2^{\frac{k}{2}}\Gamma\left(\frac{k}{2}\right)}\, x^{\frac{k}{2}-1} e^{-\frac{x}{2}}$	CDF	$\frac{1}{\Gamma\left(\frac{k}{2}\right)}\, \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$

Examples: CHDF(2, 0) = 0.5 and CHCP (2,1) => P=0.393469341

Note that for the value combination m=2 and X=0 the built-in power function Y^X will return DATA ERROR. This has been overcome in CHDF (but not in the driver program) by using an alternative version of Y^X that returns 0^0=1.

Student's T-Distribution { TSDF , TSCP , TDST }

In probability and statistics, Student's t-distribution (or simply the t-distribution) is any member of a family of continuous probability distributions that arises when estimating the mean of a normally distributed population in situations where the sample size is small and population standard deviation is unknown. It was developed by William Sealy Gosset under the pseudonym Student. Whereas a normal distribution describes a full population, t-distributions describe samples drawn from a full population; accordingly, the t-distribution for each sample size is different, and the larger the sample, the more the distribution resembles a normal distribution.



There are three functions available in the XMSTAT module dealing with this distribution.

- This first two are independent functions to calculate the density function (**TSDF**) and the CENTRAL probability (**TSCP**) – between [-x, x] - independently, taking the input parameters from Y (degrees of freedom) and X (argument). The result is placed in X but no LastX is done.

- The third one (**TDIST**) is also a driver program in the same guise as those seen previously. The menu presents the three usual choices:

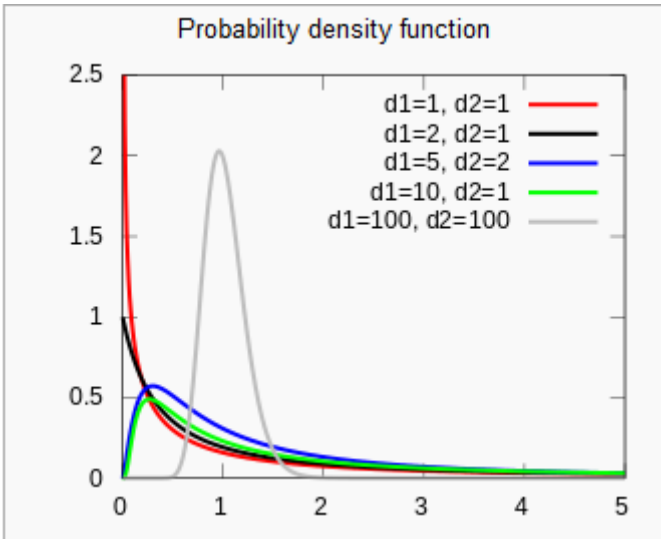
  Degrees of freedom in [**A**],
  Density function in [**B**], and
  Cumulative Probability in [**C**].

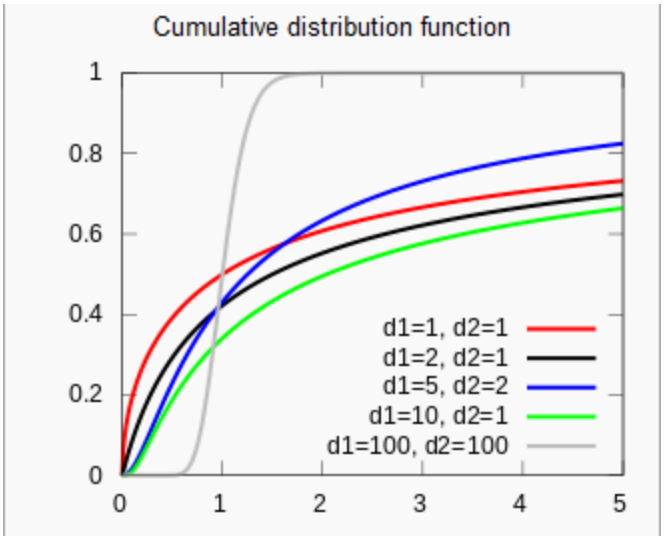
TSDF and **TSCP** are hybrid MCODE/Focal functions that don't make any use data registers, but they use the ALPHA registers and the stack. **TDIST** on the other hand is data-register hungry, you need to have SIZE 12 or higher for this routine to work correctly.

Examples: **TSDF**(4, 1) = 0.214662526; and **TSCP**(4, 1) = 0.626099034

Snedecor's F-Distribution { FSDF , FSCP , FDST }

The F-distribution, also known as Snedecor's F distribution or the Fisher–Snedecor distribution (after Ronald Fisher and George W. Snedecor) is, in probability theory and statistics, a continuous probability distribution.
If a random variable X has an F-distribution with parameters n1 and n2, we write X ~ F(n1, n2). The parameters n1 and n2 are positive integers, but the distribution is well-defined for positive real values of these parameters.





There are three functions available in the XMSTAT module dealing with this distribution.

- This first two are independent functions to calculate the density function (F**SDF**) and the TAILING probability (F**SCP**) – between [x, infinite[- independently, taking the input parameters from Z, Y (degrees of freedom) and X (argument). The result is placed in X but no LastX is done.

- The third one (**FDST**) is also a driver program in the same guise as those seen previously. The menu presents the three usual choices:

  Parameter n1 in [**A**],
  Parameter n2 in [**B**], and
  Cumulative Probability in [**C**].

FSDF and **FSCP** are hybrid MCODE/Focal functions that don't make any use data registers, but they use the ALPHA registers and the stack. For **FDST** however, you need to have SIZE 08 or higher for this routine to work correctly.

Examples: **FSDF**(7, 6, 4.3) = 0.022012383, and **FSCP**(7, 6, 4.3) = 0.047640800

Note: The implementation for the Cumulative Probability does not use the incomplete beta function but trigonometric approximations. It is therefore faster but not so accurate for small values of the argument x.

Binomial Distribution { BNP , BNP+ , BIN }

In probability theory and statistics, the binomial distribution with parameters n and p is the discrete probability distribution of the number of successes in a sequence of n independent yes/no experiments, *each of which yields success with probability p*. A success/failure experiment is also called a Bernoulli experiment or Bernoulli trial; when n = 1, the binomial distribution is a Bernoulli distribution. The binomial distribution is the basis for the popular binomial test of statistical significance.

Probability Singe-Event function

In general, if the random variable X follows the binomial distribution with parameters $n \in \mathbb{N}$ and $p \in [0,1]$, we write $X \sim B(n, p)$. The probability of getting exactly k successes in n trials is given by the probability mass function:

$$f(k; n, p) = \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Cumulative Probability function.

The cumulative Probability function can be expressed as:

$$F(k; n, p) = \Pr(X \leq k) = \sum_{i=0}^{\lfloor k \rfloor} \binom{n}{i} p^i (1-p)^{n-i}$$

where [k] is the "floor" under k, i.e. the greatest integer less than or equal to k.

There are three functions available in the XMSTAT module dealing with this distribution.

- This first two are independent functions to calculate the density function (**BNP**) for a single event and the cumulative probability (**BNP+**) for all previous events – in both cases taking the input parameters from Z, Y (n and p) and X (single or multiple event). The result is placed in X but no LastX is done. Theses routines use data registers R00 to R03.

- The third one (**BIN**) is also a driver program but contrary to the previous ones, this will prompt sequentially for the parameters and the number of events – to calculate the same value as BNP (i.e. non-cumulative case).

Note that to obtain a cumulative probability we could simply add all the single-event probabilities for those arguments less than or equal to argument. The implementation of **BNP+** however uses a different approach, which is faster that the simple addition of individual factors.

Examples: calculate BNP(100, 1/6, 15) as single-event and also cumulative:

BNP(100, 1/6, 15) = 0.100236634; and BNP+(100, 1/6, 15) = 0.387657550

Appendix. Some FOCAL Program Listings

1. Binary Distribution, by JM Baillard.

```
01  LBL "BNP+"
02  STO 00
03  SIGN
04  X<>Y
05  STO 01
06  -
07  ST/ 01
08  X<>Y
09  STO 02
10  Y^X
11  SIGN
12  RCL
00
13  X=0?
14  GTO 02
15  CLX
16  STO 03
17  LASTX
18  LBL 01
19  LASTX
20  RCL
01
21  *
22  RCL 02
23  RCL 03
24  -
25  *
26  RCL 03
27  1
28  +
29  STO
03
30  /
31  +
32  DSE 00
33  GTO
01
34  SIGN
35  LBL 02
36  LASTX
37  END
```

2. Chi-Squared Distribution, by JM Baillard

```
01  LBL "CH2"
02  STO N
03  X<>Y
04  STO O
05  2
06  ST+ Y
07  /
08  GAMMA
09  RCL N
10  2
11  /
12  E^X
13  ST* Y
14  X<> L
15  RCL
O
16  2
17  /
18  Y^X
19  X<>Y
20  /
21  STO M
22  1
23  RCL
O
24  1
25  ENTER^
26  LBL 01
27  X<> T
28  RCL N
29  *
30  R^
31  2
32  +
33  STO T
34  /
35  STO T
36  X<>Y
37  ST+ Y
38  X#Y?
39  GTO
01
40  RCL M
41  *
42  RCL N
43  SIGN
44  X<>
O
45  X<>Y
46  CLA
47  END
```

3. Normal Distribution; by "Mike (Stgt)"

```
01*LBL "XXP"
02 X<0?
03 LOG
04 1
05 +
06 2
07 /
08*LBL "XP"
09 X<=0?
10 LOG
11 STO 11
12 ,5
13 X>Y?
14 SF 18
15 SIGN
16 X<>Y
17 X>Y?
18 SIGN
19 FC? 18
20 -
21 1/X
22 X^2
23 LN
24 SQRT
25 ENTER^
26 ENTER^
27 STO O
28 1308 E-6
29 *
30 ,189269
31 +
32 *
33 1,432788
34 +
35 *
36 1
37 +
38 X<> O
39 ,010328
40 *
41 ,802853
42 +
43 *
44 2,515517
45 +
46 RCL O
47 /
48 -
49 FS?C 18
50 CHS
51 "GI"
52 ASTO 06
53  E-3
54 X<>Y
55 SF 10
56 XROM "SV"
57 CF 10
58 RTN
59*LBL "PXX"
60 ABS
61 SF 18
62*LBL "PX"
63 ABS
64 6
65 X>Y?
66 GTO 09
67 LASTX
68 SIGN
69 X<0?
70 CLX
71 CF 18
72 RTN
73*LBL 09
74 RCLFLAG
75 STO 10
76 FIX 7
77 LASTX
78 ENTER^
79 X^2
80 STO N
81 SIGN
82 STO O
83 X<>Y
84 STO M
85 ABS
86*LBL 01
87 LASTX
88 RCL N
89 RCL O
90 2
91 +
92 STO O
93 /
94 *
95 ST+ M
96 RND
97 X#0?
98 GTO 01
99 RCL 10
100 STOFLAG
101 RCL N
102 2
103 /
104 CHS
105 E^X
106 PI
107 ST+ X
108 SQRT
109 /
110 RCL M
111 *
112 ENTER^
113 FC?C 18
114 ,5
115 +
116 FC? 10
117 RTN
118 RCL 11
119 -
120 END
```

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